Determination of the Parameters of Cavities Terminating Transmission Lines

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Introduction

PAST methods available for measuring parameters of cavities terminating transmission lines are often considered too involved for production line testing.[1,2] One common method requires many individual standing wave ratio measurements at frequencies slightly lower and higher than the resonant frequency of the cavity. These measurements take not only an excessive length of time but also require an accurately controlled variable frequency oscillator for precision measurements. A second method in which an oscillographic display of reflected power is used, while much faster, requires more complicated equipment and calls for use of unconventional measurement procedures.

The method presented herein was devised to meet time and accuracy requirements on the production line. Its speedy results can be obtained from equipment and calculations with which production line personnel are familiar. Use is made of the same common sweep frequency method utilized in testing transmission and absorption cavities. This paper presents a derivation of the theory of the method, a description of the apparatus used, and a comparison of the results obtained with this and the point by point method.

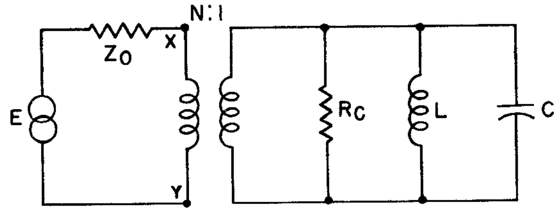

Fig. 1—Simplified equivalent circuit of cavity.

Theory

Cavity Coupling Factor

An equivalent circuit of a cavity that terminates a radio frequency transmission line of characteristic impedance Z_0 is illustrated in Fig. 1. Here, it is assumed that there is no line loss, no resistive loss in the coupling mechanism between the cavity and the line, and that the generator is matched to the line. The points labeled X and Y are considered to be the points on the transmission line at which the cavity presents the impedance Z. The impedance looking into the cavity is

$$Z = \frac{N^2}{\frac{1}{R_c} + j\left[\omega c - \frac{1}{\omega L}\right]} \tag{1}$$

where

R_c = equivalent cavity shunt resistance
L = equivalent cavity shunt inductance
C = equivalent cavity shunt capacitance
N = equivalent transformation between line and cavity.

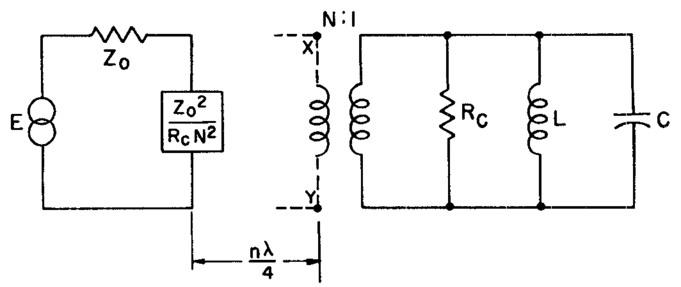

Fig. 2—Equivalent impedance of cavity at resonance at point on line one-quarter wavelength from cavity.

At resonance, $\omega c = 1/\omega L$ and $Z = R_c N^2$. If the voltage across the line is measured at a point on the line an odd multiple of one-quarter wavelengths from the cavity (see Fig. 2), the voltage is

$$E_1 = \frac{\frac{Z_0^2}{R_c N^2} E}{Z_0 + \frac{Z_0^2}{R_c N^2}}. \tag{2}$$

Far from resonance, $Z = 0$ and the impedance across the line an odd multiple of one-quarter wavelengths from the cavity is infinite. In this case, the voltage measured across this point on the line is equal to E.

The ratio of the voltage far from resonance to that at resonance is

$$R = \frac{E}{E_1} = \frac{R_c N^2}{Z_0} + 1. \tag{3}$$

The cavity coupling factor—commonly symbolized as β—is used in specifying the performance of line terminating cavities. It is defined as the normalized cavity impedance at resonance or

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[1] C. G. Montgomery, "Technique of Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 334–336; 1947.

[2] E. D. Reed, "A sweep frequency method of Q measurement for single-ended resonators," *Proc. NEC*, vol. 7, pp. 162–172; 1951.

$$\beta = \frac{R_c N^2}{Z_0}. \tag{4}$$

By substituting β for $R_c N^2/Z_0$ in (3) and rearranging, there results

$$\beta = R - 1. \tag{5}$$

Cavity Q Factor

If it were possible to measure the voltage at the cavity itself, its loaded Q may be found from

$$Q_L = \frac{f_0}{2\Delta f} \tag{6}$$

where f_0 = the resonant frequency of the cavity, Δf = the difference between the resonant frequency and the frequency at which the voltage is .707 of that at resonance. Since it is usually impossible to measure the voltage at the cavity, it may be measured at multiples of a half-wavelength from the cavity.

If the cavity Q is high and the distance between the point of measurement and the cavity is small, negligible error will result. As the distance from the cavity increases and as the Q becomes lower, an error is introduced. This error is caused by the fact that the measurement point is only a multiple of a half-wavelength at one discrete frequency. Thus if the point is exactly a multiple of one-half wavelength for the resonant frequency, it will be slightly more or less for the .707 voltage points. Under these conditions, a correction term must be added to the impedance of the cavity when it is off resonance.

To simplify the expressions, normalized admittance will be used and N will be assumed to be unity. The admittance of the cavity near resonance may be written as

$$Y = G_c + jG_c Q_0 \frac{\Delta f}{f_0} \tag{7}$$

and the equivalent circuit of the cavity may be shown as in Fig. 3(a) and 3(b).

At resonance the cavity admittance reduces to $Y = G_c$ and Fig. 3(a) applies at points a multiple of half-wavelength from the cavity.

Near resonance at points a multiple a half-wavelengths from the cavity Fig. 3(d) applies. The correction term[3] added due to the small shift in wavelength is

$$dY = jn\pi\left(\frac{\lambda_g}{\lambda_g}\right)^2 (1 - [jy]^2) \tag{8}$$

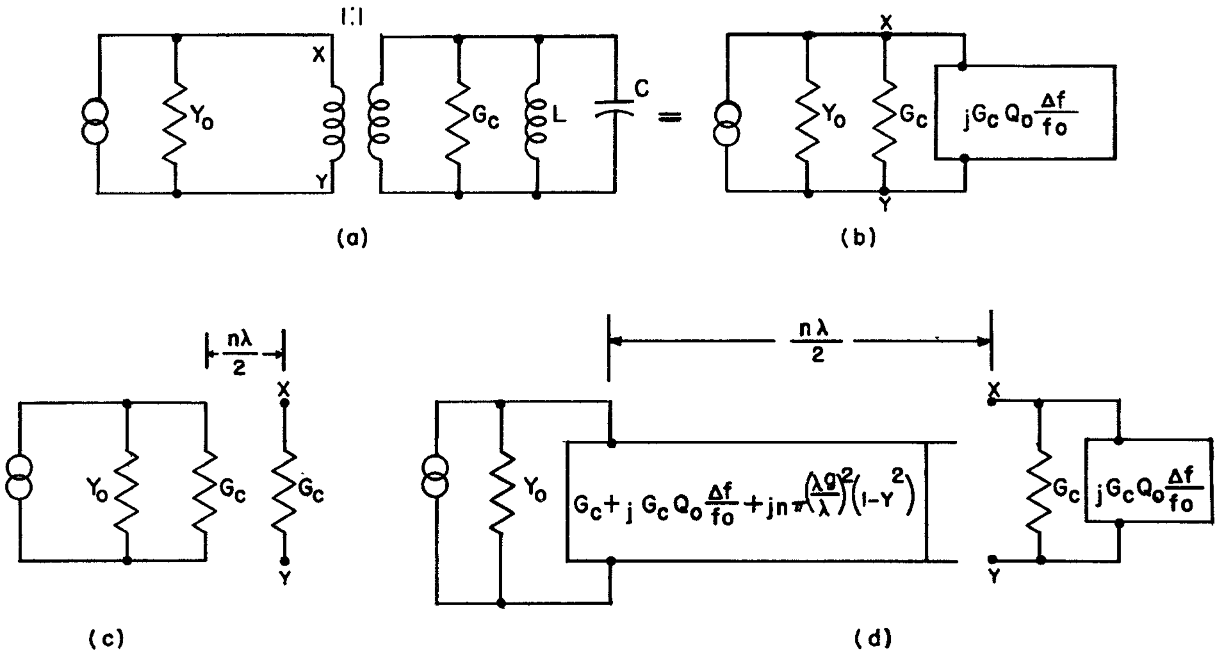

Fig. 3—Equivalent circuit of cavity in terms of admittance.

where

G_c = the conductance of the cavity
n = the number of half wavelengths between the point of measurement and the cavity
λ_g = guide wavelength
λ_0 = free space wavelength.

If the magnitude of the voltage at a frequency Δf from resonance is set equal to $\sqrt{2}/2$ times its magnitude at resonance, there results

$$\frac{.707}{Y_0 + G_c} = \frac{1}{Y_0 + G_c + jG_c Q_0 \frac{\Delta f}{f_0} + jn\pi\left(\frac{\lambda_g}{\lambda_0}\right)^2 (1 - Y^2)} \tag{9}$$

[3] N. Marcuvitz, "Waveguide Handbook," McGraw-Hill Book Co., Inc., New York, N. Y., p. 13; 1951.

solving for Q^L there finally results

$$Q_L = \frac{f_0}{2\Delta f}\sqrt{1 - 2n\pi\left(\frac{\lambda^g}{\lambda_0}\right)^2\left(\frac{1+\beta}{\beta}\right)\frac{1}{Q_L} - n^2\pi^2\left(\frac{\lambda^g}{\lambda_0}\right)^4\left(\frac{1+\beta^2}{\beta^2}\right)\frac{1}{Q_L^2}}\,. \tag{10}$$

Discussion

The equipment used for measurement is very similar to that described by Montgomery,[4] except that a slotted section and probe are placed before the cavity. The output of this probe is used for the vertical deflection of the oscilloscope. The law of the crystal detector must be taken into account when determining the various measured voltages.

The coupling parameter is found by presenting a complete klystron mode on the oscillograph screen, and adjusting the probe in the slotted section to a position a multiple of quarter-wavelengths from the point in the line at which the cavity is effectively a short circuit. This probe position produces a maximum mode height (Fig. 4(a)). The cavity is then tuned so that its dip lies on the center of the mode. The value of R is obtained as shown in Fig. 4(b).

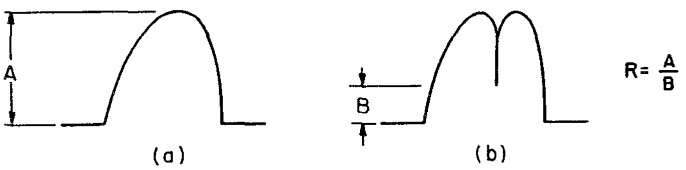

Fig. 4—Oscilloscope patterns obtained when measuring the cavity coupling parameter.

The cavity loaded Q is found by adjusting the probe position to a multiple of half-wavelengths from the cavity. This position is obtained when the mode height on the oscilloscope is made a minimum with the cavity off resonance. The cavity is then tuned to produce a cavity band pass characteristic at the center of the mode. Alternately, if the β measurement had been made first and the cavity and oscillator were set to the correct frequency the probe position may be varied to produce a symmetrical band pass characteristics on the oscilloscope (Fig. 5). The loaded Q of the cavity is then obtained by measuring 2 Δf by any of the well known methods.

The following assumptions were made in deriving the theory of these measurements:

1) Zero line loss between cavity and point of measurement
2) Zero coupling loss
3) High cavity Q factor
4) Negligible probe or voltmeter error and loading
5) Matched generator.

[4] Montgomery, *op. cit.*, p. 396.

At microwave frequencies assumptions 1, 2, 3, and 4 are usually true and 5 can be accomplished by tuning or by using matched attenuators. Hence, the errors in the measurements due to these factors are negligible for production line testing.

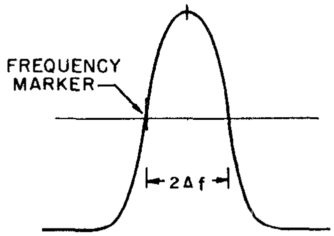

Fig. 5—Oscilloscope pattern obtained when measuring the cavity Q factor.

Tests were made on various cavities at X and C bands with this method and the point by point VSWR method. The apparatus available for the tests by the VSWR method allowed measurement of 2Δf to no better than ±0.8 mc/sec. The results obtained by both methods compare quite well. Table I gives a comparison of the results obtained on C band cavities when the probe was set at a position three half-wavelengths from the cavity.

TABLE I

Comparison of Results by Two Methods

Cavity	Simplified method		Point by point vswr method	
	β	Q_L	β	Q_L
No. 1	0.48	11300	0.50	14000
No. 2	1.00	11300	1.01	11300
No. 3	1.11	6900	1.13	6900
No. 4	0.74	5400	0.74	5650
No. 5	2.16	2350	2.20	2350

A cavity with a loaded Q of 2700 and a coupling parameter of 1.4 was tested for Q at a probe position 9/2 wavelengths from the cavity and at a second point 3/2 wavelengths from the cavity. The difference in measured Q between these two points was $1\frac{1}{4}\% \pm \frac{1}{4}\%$ per wavelength. The theoretical correction using equation 10 is $1\frac{1}{8}\%$. There is good correlation between the measured and theoretical corrections both of which are low. Thus, for most production line measurements the application of the correction is unnecessary, and the loaded Q is obtained from (6).

Acknowledgment

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